Wax
16749

ARPA NETWORK INFORMATION CENTER
Stanford Research Institute
Menlo Park, California 94025

L. Kleinrock
F. Tobagi
May, 1973

NIC # 16749
Packet Radio Temporary Note # 57

EFFECT OF ACKNOWLEDGEMENT TRAFFIC ON CHANNEL THROUGHPUT IN PACKET RADIO SYSTEMS

1. INTRODUCTION

It has always been considered that acknowledgement traffic, in packet radio systems, was carried by a separate channel so that it never interfered with information traffic. In this note we are concerned with the degradation in throughput due to acknowledgement traffic when the latter is carried by the same channel.

The problem of channel overhead due to the error control scheme used was already attacked by R. Binder and L. Castonguay [B1]. However, in their analysis, a two-channel system was assumed: an Aloha Channel from terminals to station and a queued channel from station to terminals. This situation is actually encountered in the ALOHA SYSTEM.

In packet radio systems, however, a single channel is generally assumed. In such environments, acknowledgement traffic flows in both directions, and there is a strong correlation between information traffic and acknowledgement traffic rendering the analysis of [B1] inapplicable.

Throughout the note, only the conventional positive acknowledgement-retransmit (POSACK) error control scheme is assumed. Also, it is used on a hop-by-hop basis, so that the analysis can be reduced to a single hop system. Furthermore, a zero processing time is assumed at receivers, i.e., if an information packet is correctly received, a POSACK is immediately generated and transmitted.

In this note we analyze the throughput (in terms of information traffic) of slotted Aloha and carrier sense access modes with acknowledgement traffic on the same channel; numerical results are discussed in section 4. In section 5 we address ourselves to the fundamental question: single channel vs. separate channels.

2. NOTATION

T = information packet transmission time

T_a = acknowledgement packet transmission time

$$\alpha = \frac{T_a}{T}$$

τ = propagation delay

$$a = \frac{\tau}{T}$$

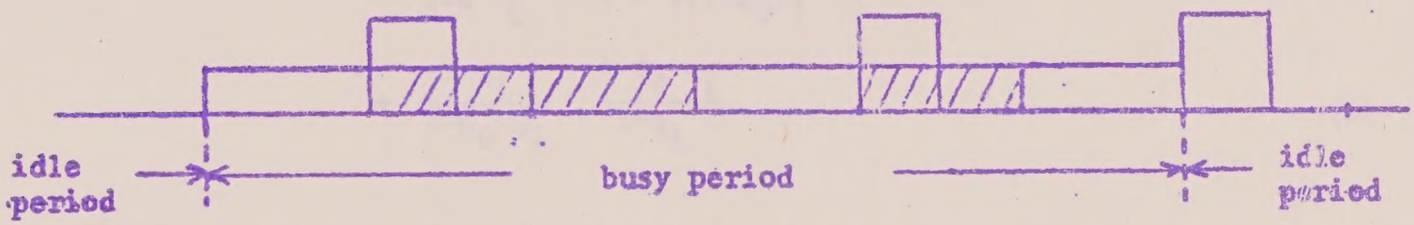
3. ANALYSIS

3.1 Slotted Aloha

In slotted systems, it is reasonable to assume $a = 1$. A packet will not need further retransmissions if and only if it is correctly received by the receiver and its POSACK is correctly received back. Two slotted systems are considered. The first assumes no cooperation between the transmitters; information packets can interfere with acknowledgement packets. The second assumes that all transmitters (considered to be within range), with a packet ready for transmission, perform the error-check on all packets that are transmitted: if the transmission is successful, then terminals are prohibited from transmitting their packet (knowing that an ACK will be transmitted) in the following slot. In this second system, acknowledgement packets are always successful.

The cycle analysis often used previously is once again applied here.

3.1.1 Slotted Aloha Without Terminal Cooperation



Disregarding ACK packets, we can define a busy period, an idle period and a cycle in much the same way as before [K1, K2].

Since we require the packet and its acknowledgement to be received correctly, only one packet (the last in a busy period) may contribute to the throughput since it is the only packet whose acknowledgement packet does not incur any interference.

A busy period will contain m information packets with probability

$$q_0(1-q_0)^{m-1} \tag{1}$$

where $q_0 = e^{-G}$ (2)

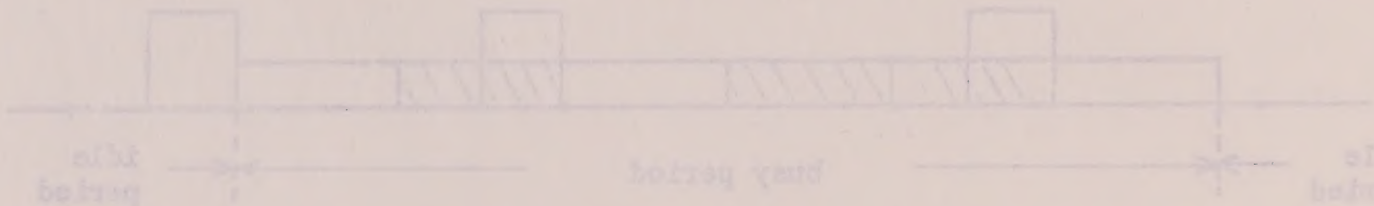
Let P_s^i denote the probability of success of the ith information packet in a busy period.

We have:

$$P_s^1 = \frac{q_1}{1-q_0} \tag{3}$$

$$q_1 = Ge^{-G} \tag{4}$$

and $$P_s^i = (1-P_s^{i-1}) \frac{q_1}{1-q_0} \tag{5}$$



Let $c = \frac{q_1}{1-q_0}$. Eq. (5) can be written as

$$p_s^i = c - cp_s^{i-1} \tag{6}$$

and its solution is

$$p_s^m = P_r\{\text{success at the mth slot}\} = \frac{c[1-(-c)^m]}{1+c} \tag{7}$$

Removing the condition on m, we get:

$$\begin{aligned}\bar{U} &= P_r\{\text{success during the last slot of a busy period}\} \\ &= \sum_{m=1}^{\infty} p_s^m q_0(1-q_0)^{m-1} \\ &= \frac{c}{1+c}\left[1 + \frac{q_0}{1+c(1-q_0)}\right] \\ &= \frac{q_1}{1-q_0+q_1} \cdot \frac{1+q_0+q_1}{1+q_1}\end{aligned} \tag{8}$$

since

$$\bar{B} = \frac{1}{q_0} \tag{9}$$

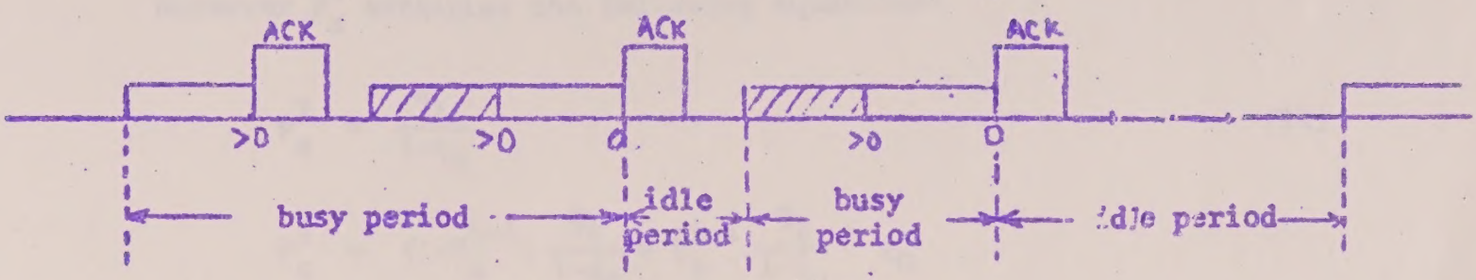
and

$$\bar{I} = \frac{1}{1-q_0} \tag{10}$$

The throughput can be expressed as:

$$\begin{aligned}S &= \frac{\bar{U}}{\bar{B}+\bar{I}} \\ &= \frac{Ge^{-2G}(1-e^{-G})(1+e^{-G}+Ge^{-G})}{(1+Ge^{-G})(1-e^{-G}+Ge^{-G})}\end{aligned} \tag{11}$$

3.1.2 Slotted Aloha With Terminal Cooperation



In this system, the idea is to give priority to the acknowledgement packets. To do so, the devices are considered to be able to perform the following operations:

+ they have the capability of performing error checks on any slot

+ they can distinguish between an information packet and an acknowledgement packet.

A terminal, involved in a conversation, performs error-checks over each slot. If the generation (or rescheduling) of its packet takes place in a slot during which one of the following is true:

1 - no packets were transmitted

2 - an acknowledgement packet was transmitted

3 - the information packet transmitted does not have a correct sum-check,

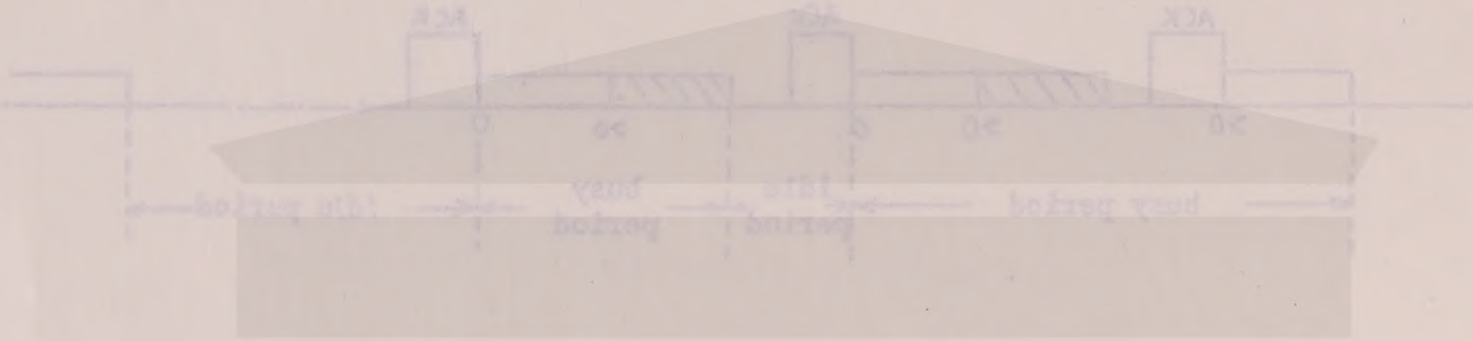
then the terminal transmits its packet over the next slot (as in normal operation of slotted Aloha).

If the generation (or rescheduling) of the packet takes place in a slot during which an information packet with a correct sum-check is received, then the terminal delays the transmission of its packet by one slot (to avoid conflict with the ACK packet).

The busy period:

If some (non-zero) number of packets arrive to the channel during a slot, then the same number of packets are accumulated at the beginning of the next slot, which we call a non-zero-slot. By the same token, we define a zero-slot.

Define the busy period as the time lapse between a non-zero-slot immediately following a zero-slot, and the first zero-slot encountered (excluding the latter).



Digitized by the Internet Archive
in 2025 with funding from
Amateur Radio Digital Communications, Grant 151

https://archive.org/details/temporary-note-057

With this definition, a busy period consists of m information packets with probability $q_0(1-q_0)^{m-1}$.

Moreover P_s^i satisfies the following equations:

$$P_s^1 = \frac{q_1}{1-q_0} \tag{12}$$

$$P_s^i = (1-P_s^{i-1}) \frac{q_1}{1-q_0} + P_s^{i-1} \frac{q_1}{1-q_0} \cdot q_0$$

$$= \frac{q_1}{1-q_0} - q_1 P_s^{i-1} \tag{13}$$

Let $c' = -q_1$. (14)

We have

$$P_s^i = \frac{c}{1-c'} [1-c'^i] . \tag{15}$$

Let $\bar{A}_m$ be the average time, during a busy period of m information slots, occupied by acknowledgement packets; we have:

$$\bar{A}_m = \sum_{i=1}^{m-1} P_s^i$$

$$= \frac{c}{1-c'} \left[m-1 - \frac{c'[1-c'^{m-1}]}{1-c'} \right] \tag{16}$$

(The summation is taken up to m-1 since, according to our definition of the busy period, the ack. corresponding to the last packet in the busy period is part of the idle period.)

Let $\bar{U}_m$ be the average time, during a busy period of m information slots, occupied by successful packets; we have:

$$\bar{U}_m = \sum_{i=1}^{m} P_s^i = \frac{c}{1-c'} \left[m - \frac{c'[1-c'^m]}{1-c'} \right] \tag{17}$$

Removing the condition on m we get:

$$\bar{A} = \sum_{m=2}^{\infty} \bar{A}_m q_0 (1-q_0)^{m-1} =$$

$$= \frac{c}{1-c'}\left[\frac{1}{q_0} - 1 - \frac{c'}{1-c'}\left(1 - \frac{q_0}{1-c'(1-q_0)}\right)\right]$$

$$= \frac{G}{1 + Ge^{-G} - Ge^{2G}} \qquad (18)$$

and $$\bar{U} = \frac{c}{1-c'}\left[\frac{1}{q_0} - \frac{c'}{1-c'}\left(1 - \frac{q_0 c'}{1-c'(1-q_0)}\right)\right]$$

$$= \frac{G}{(1-e^{-G})(1+Ge^{-G}-Ge^{-2G})} \qquad (19)$$

The throughput can then be written as:

$$S = \frac{\bar{U}}{\frac{1}{q_0} + \bar{A} + \frac{1}{1-q_0}}$$

$$= \frac{Ge^{-G}}{1 + 2Ge^{-G}(1-e^{-G})} \qquad (20)$$

3.2 Carrier-sense

There are, a priori, two advantages in carrier-sense:

(i) The implementation of priority traffic is much simpler.

(ii) Multiple-size packets are handled very naturally, so that it makes it reasonable to consider $\alpha \neq 1$ (normally $\alpha < 1$).

The analysis here uses the same techniques as in previous sections. However, in order to keep the analysis tractable, some approximations are made.

System operation

To give priority to acknowledgement traffic, the system should operate as follows:

(i) If a terminal, with a packet ready for transmission, senses the carrier idle, then the terminal transmits its packet τ seconds later if and only if the channel is still sensed idle.

(ii) If such a terminal senses the channel busy, then it waits until the channel gets idle and operates as described in (i).

(iii) All acknowledgement packets are transmitted right away, without incurring the τ sec. delay.

Referring to note #24 [K1], y is the time of occurrence of the last packet arriving in the first a sec. of a packet transmission.

The distribution of y was derived in the appendix of note 24.

In the present note, for the sake of simplicity, the analysis assumes a constant y, denoted by $\bar{y}$. Two cases can be considered:

1) $\bar{y} = a$ (yields a lower bound on throughput),

2) $\bar{y} = a - \frac{1}{G}(1-e^{-aG})$ (expected value of y) . (21)

Furthermore one can use an upper bound on (21) using

$$\bar{y} = \frac{a^2 G}{2} \tag{22}$$

The last one is retained for the numerical results.

Figure 1 illustrates the behaviour of the channel under the above operating mode. Also from Fig. 1, one can easily define a transmission period (TP), and an acknowledgement period (AP).

Under the approximations stated above, we have:

$$|TP| = 1 + 2a + \bar{y} \tag{23}$$

$$|AP| = a + a \tag{24}$$

Every successful TP (S-TP) is followed by an AP. Thus, the channel looks like a sequence of TP's, AP's, and eventually some "quiet" periods (QP) (see Figure 2).

At the end of each TP, some number of packets accumulate. The distribution of this number is Poisson with mean $G(1+a+\bar{y})$.

Let $$q_0 = e^{-G(1+a+\bar{y})} \tag{25}$$

$$q_1 = G(1+a+\bar{y}) \cdot e^{-G(1+a+\bar{y})} \tag{26}$$

A TP at the end of which no packets accumulated is called a zero-TP (Z-TP). By the same token, we define a non-zero-TP (NZ-TP).

Define a busy period as the time lapse starting with the first TP following a Z-TP up to and including the first Z-TP encountered. The time separating two consecutive busy periods is called an idle period.

Figure 3 illustrates the various situations of idle periods encountered.

The distribution of number of TP in a busy period is geometric with parameter $1-q_0$, i.e.,

$$P_r\{m \text{ TP's in a busy period}\} = q_0(1-q_0)^{m-1} \tag{27}$$

Probability of success over a TP

The existence of several situations encountered for an idle period renders the probability of success over the first TP in a busy period, P_s^1, dependent on the way that busy period started. Let P_s^1 be, for the time being, an unknown.

For i>1 we have:

$$p_s^i = (1-p_s^{i-1}) \frac{q_1}{1-q_0} e^{-aG} + p_s^{i-1} \frac{q_1}{1-q_0} \cdot e^{-G(\alpha+a)} e^{-aG}$$

$$= \frac{q_1}{1-q_0} e^{-aG} + \frac{q_1}{1-q_0} e^{-aG} (e^{-G(\alpha+a)} - 1) p_s^{i-1} \qquad (28)$$

Let $$c = \frac{q_1}{1-q_0} e^{-aG} \qquad (29)$$

$$c' = \frac{q_1}{1-q_0} e^{-aG} (e^{-G(\alpha+a)} - 1) \qquad (30)$$

Eq. (28) reduces to

$$p_s^i = c + c' p_s^{i-1} \qquad i > 1 \qquad (31)$$

whose solution is given in terms of P_s^1 by:

$$p_s^i = c \left[\frac{1-c'^{i-1}}{1-c'} \right] + c'^{i-1} p_s^1 \qquad (32)$$

Let P_s^{last} denote the probability of success over the last TP in a busy period. We have:

$$p_s^{last} = \sum_{m=1}^{\infty} p_s^m q_0 (1-q_0)^{m-1}$$

$$= \frac{c}{1-c'} \left[1 - \frac{q_0}{1-c'(1-q_0)} \right] + \frac{q_0}{1-c'(1-q_0)} p_s^1 \qquad (33)$$

$$= A + B P_s^1 \qquad (34)$$

with

$$B = \frac{q_0}{1-c'(1-q_0)} \qquad (35)$$

$$A = \frac{c}{1-c'} [1-B] \qquad (36)$$

By examining the three situations illustrated in Figure 3, we can write another relation between P_s^1 and P_s^{last} as follows:

$$P_s^1 = (1-P_s^{last})e^{-aG} + P_s^{last}\left[e^{-G(\alpha+a)} \cdot e^{-aG} + G(\alpha+a)e^{-G(\alpha+a)} \cdot e^{-aG}\right]$$

$$= e^{-aG} + e^{-aG}\left[e^{-G(\alpha+a)}(1+aG+\alpha G) - 1\right] P_s^{last} \quad (37)$$

$$= C + D\, P_s^{last} \quad (38)$$

with $C = e^{-aG}$

$$D = e^{-aG}\left[e^{-G(\alpha+a)}(1+aG+\alpha G) - 1\right] \quad (39)$$

From Eq. 34 and Eq. 38 we solve for P_s^1:

$$P_s^1 = \frac{C + A \cdot D}{1 - D \cdot B} \quad (40)$$

Average number $\bar{U}$ of S-TP in a busy period

Conditioning on m TP in a busy period we have

$$\bar{U}_1 = P_s^1 \quad (41)$$

$$\bar{U}_m = P_s^1 + \sum_{i=2}^{m} P_s^i \qquad m > 1$$

$$= P_s^1 + \frac{c}{1-c'}\left[m - 1 - \frac{c'(1-c'^{m-1})}{1-c'}\right] + \frac{c'(1-c'^{m-1})}{1-c'} P_s^1 \quad (42)$$

Removing the condition on m we have:

$$\bar{U} = \sum_{m=1}^{\infty} \bar{U}_m\, q_0\, (1-q_0)^{m-1}$$

$$= P_s^1 + \frac{c}{1-c'}\left[\frac{(1-q_0)}{q_0} - \frac{c'}{1-c'}\left(1 - q_0 - \frac{c'q_0(1-q_0)}{1-c'(1-q_0)}\right)\right]$$

$$+ \frac{c'}{1-c'} P_s^1 \left(1 - q_0 - \frac{c'q_0(1-q_0)}{1-c'(1-q_0)}\right) \quad (43)$$

Average number $\bar{A}$ of AP in a busy period

$$\bar{A}_1 = 0 \tag{44}$$

$$\bar{A}_m = p_s^1 + \sum_{i=2}^{m-1} p_s^i \qquad m > 1$$

$$= p_s^1 + \frac{c}{1-c'}\left[m - 2 - \frac{c'(1-c'^{m-2})}{1-c'}\right] + \frac{c(1-c'^{m-2})}{1-c'} p_s^1 \tag{45}$$

$$\bar{A} = \sum_{m=2}^{\infty} \bar{A}_m q_0 (1-q_0)^{m-1}$$

$$= p_s^1(1-q_0) + \frac{c(1-q_0)}{1-c'}\left[\frac{(1-q_0)}{q_0} - \frac{c'}{1-c'}\left(1 - \frac{q_0}{1-c'(1-q_0)}\right)\right]$$

$$\frac{c'(1-q_0)}{1-c'}\left(1 - \frac{q_0}{1-c'(1-q_0)}\right) p_s^1 \tag{46}$$

Average busy period

$$\bar{B} = \frac{1}{q_0}(1 + 2a + \bar{y}) + \bar{A}(\alpha + a) \tag{47}$$

Average idle period

$$\bar{I} = (1-p_s^{last})\frac{1}{G} + p_s^{last}\left[e^{-G(\alpha+a)}\left(\alpha + a + \frac{1}{G}\right) + \left(1 - e^{-G(\alpha+a)}\right)(\alpha + a)\right]$$

$$= (1-p_s^{last})\frac{1}{G} + p_s^{last}\left[\alpha + a + e^{-G(\alpha+a)} \cdot \frac{1}{G}\right] \tag{48}$$

where p_s^{last} is given by:

$$p_s^{last} = \frac{A + B \cdot C}{1 - B \cdot D} \tag{49}$$

Throughput

$$S = \frac{\bar{U}}{\bar{B} + \bar{I}} \tag{50}$$

4. Numerical results

Numerical results corresponding to the slotted Aloha systems are summarized in Fig. 4. The maximum attainable throughput is .26 for the slotted system with terminal cooperation (Eq. 20) and only .15 for the slotted system without terminal cooperation (Eq. 11).

In Fig. 5 we plot, for the carrier-sense access mode, the throughput as a function of channel traffic for various values of a (0, 0.1, 0.2) and α (1, .5, .25, .1). Note that the overhead due to acknowledgement traffic can be sensitive to α. This is illustrated in Fig. 5 and the following table:

a \ α	1	.5	.25	.1
0	.315	.396	.455	.50
0.1	.262	.316	.351	.377
0.2	.223	.261	.284	.3

Table 1. Maximum throughput for various values of α and a.

5. Single channel vs. separate channels

5.0 The idea here is to compare, in terms of information throughput, the following, with the total available capacity being held constant:

(i) a single channel for both information and acknowledgement packets

(ii) separate channels: one for information packets and one for acknowledgement traffic.

For (i), of course the total capacity is used for the single channel. For (ii), the total capacity is optimally split between the two channels.

For the separate channel approach, two different systems were considered:

a - The first consists of sending the acknowledgement packets as soon as they are generated. The capacity assigned to the acknowledgement channel is sufficient to avoid overlap between any two consecutive acknowledgement packets. (It will be called separate channel without queueing.)

b - In the second system,* we assume the existence of a device whose sole role is to perform the error checks on the transmitted packets and to issue the positive acknowledgements when the sum-check is correct. Such a station can queue the packets, does not interfere with itself, and requires less capacity for the ACK channel than the previous system. (It will be called separate channels with queueing.)

5.1 Notation

C	total capacity available
C_1	capacity of information channel in separate channels case
C_2	capacity of acknowledgement channel in the separate channels case
B	number of bits per packet
αB	number of bits per acknowledgement packet
S_{max}	maximum throughput in packets per packet transmission time
λ_{max}	maximum throughput in packets per second

5.2 Slotted Aloha system

For the single channel case we have

$$\lambda_{max} = S_{max} \cdot \frac{C}{B} \tag{51}$$

where S_{max} equals .26 or .15 (see Figure 4).

Remark 1: Recall that in this case we assume that an acknowledgement packet will use up the whole slot even though $\alpha < 1$.

For the separate channels case without queueing, we require:

$$C_1 + C_2 = C \tag{52}$$

and

$$C_2 = \alpha C_1 \tag{53}$$

---

* Note that the environment we are focusing on is a large population of terminals and other devices all within range. Furthermore errors due to random noise are considered by far negligible compared to errors due to multiaccess interference.

therefore $$C_1 = \frac{C}{1+\alpha} \tag{54}$$

$$C_2 = \frac{\alpha C}{1+\alpha} \tag{55}$$

and $$\lambda_{max} = \frac{S_{max} C_1}{B}$$

$$= \frac{S_{max}}{1+\alpha} \cdot \frac{C}{B} \tag{56}$$

where S_{max} is 1/e (slotted Aloha).

For the separate channels case with queueing, we require:

$$C_1 + C_2 = C$$

$$C_2 = S_{max} \cdot \frac{C_1}{B} \cdot \alpha B \tag{57}$$

Remark 2: It is important to mention here that we are assigning to the acknowledgement channel the minimum capacity required, and that this results in infinite delays and upper bounds on λ_{max}.

Therefore $$C_1 = \frac{C}{1 + \alpha S_{max}} \tag{58}$$

$$C_2 = \frac{\alpha S_{max}}{1 + \alpha S_{max}} C \tag{59}$$

and $$\lambda_{max} = \frac{S_{max} C_1}{B}$$

$$= \frac{S_{max}}{1 + \alpha S_{max}} \cdot \frac{C}{B} \tag{60}$$

where, here too, $S_{max} = 1/e$.

In Fig. 6 we plot λ_{max} vs. α for the systems above. Note that the case of separate channels with queueing always perfoms better than all others.

Single channel with terminal cooperation is worse than separate channels without queueing for α <.43. This is due to Remark 1.

5.3 Carrier sense systems

For a given packet size and a given propagation delay τ, the maximum throughput attainable is a function of a and hence of the capacity of the channel, which we denote by $S_{max}(a)$.

We have

$$a = \frac{\tau \cdot C}{B} \tag{61}$$

For the single channel case, $S_{max}(a)$ is obtained from Eq. 50

Separate channels without queueing:

Equations 52, 53, 54, and 55 are still valid. Eq. 56 is now expressed as

$$\lambda_{max} = \frac{S_{max}\left(\frac{\tau}{B} \cdot \frac{C}{1+\alpha}\right)}{1 + \alpha} \cdot \frac{C}{B} \tag{62}$$

where $S_{max}(a)$ is given by Fig. 2 of PRT note #24 [K1].

Separate channels with queueing:

If C_1 is the capacity assigned to the information channel, its maximum throughput is given by $S_{max}\left(\frac{\tau}{B} C_1\right)$. C_2 must then satisfy

$$C_2 = S_{max}\left(\frac{\tau}{B} C_1\right) \alpha C_1 \tag{63}$$

From Eq. 52 we have

$$C = C_1 \left[1 + \alpha S_{max}\left(\frac{\tau}{B} C_1\right)\right] \tag{64}$$

where $S_{max}(a)$ is, here too, given by Fig. 2 of PRT note #24 [K1].
For each value of α, Eq. 64 is solved iteratively.

Numerical results are summarized in Figures 7 and 8. Figure 7 corresponds to the case $(\tau/B)\cdot C=10^{-2}$ (for example, $\tau=50\mu s.$, B=500 bits/packet, C=100 KB/S). Figure 8 corresponds to the case $(\tau/B)\cdot C=.1$ (same values for τ and B, C= 1 MB/S).

Note that one expects the three systems to be equivalent for the limiting case $\alpha=0$. The difference, however, is due to the fact that the implementation of priority traffic in the carrier sense access mode is done at the expense of forced τ periods delays.

REFERENCES

[B1] Binder, R. and Castonguay, L., "Acknowledgement Schemes in an Aloha Channel," Packet Radio Temporary Note #44, NIC #15687, April 13, 1973.

[K1] Kleinrock, L. and Tobagi, F. "Throughput in Carrier Sense Auto Slot) Packet Radio Systems," Packet Radio Temporary Note #24, NIC 14212, February 12, 1973.

[K2] Kleinrock, L., and Tobagi, F. "Carrier Sense with Initial Random Transmission Delay," Packet Radio Temporary Note #37, NIC 15334, March, 1973.

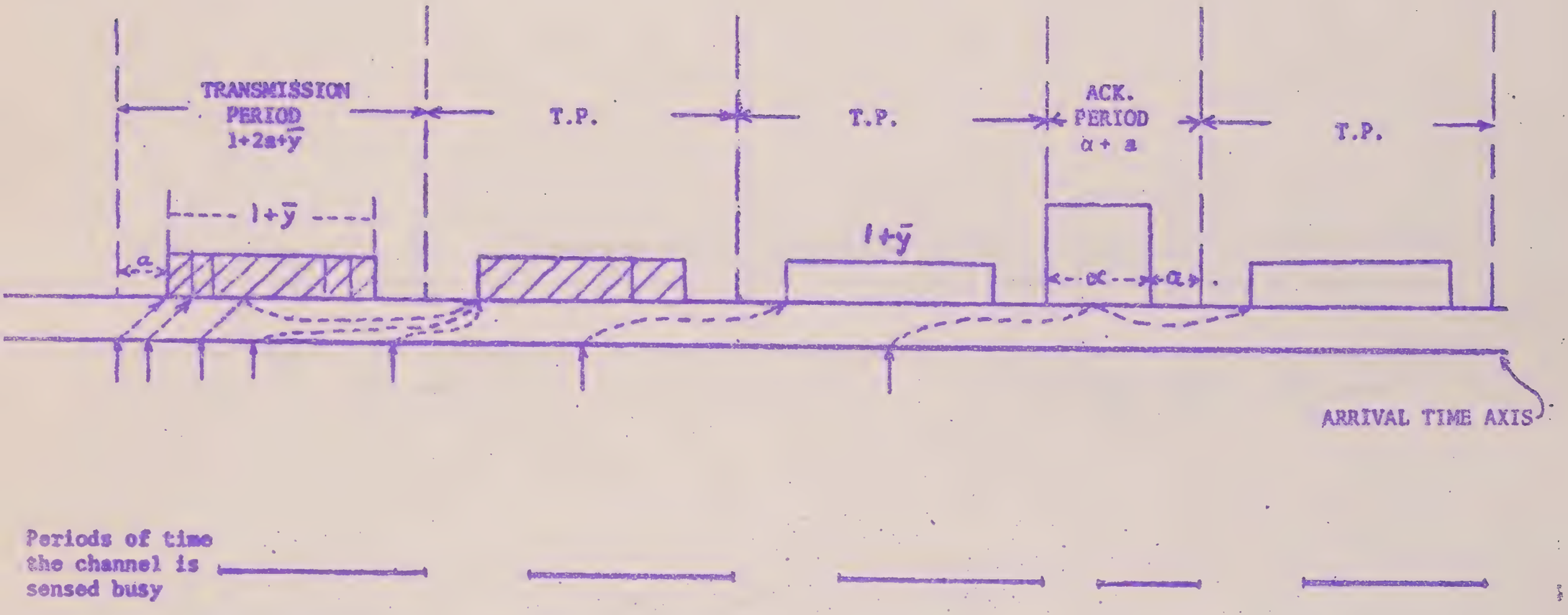


Fig. 1. Carrier sense with priority acknowledgement traffic.

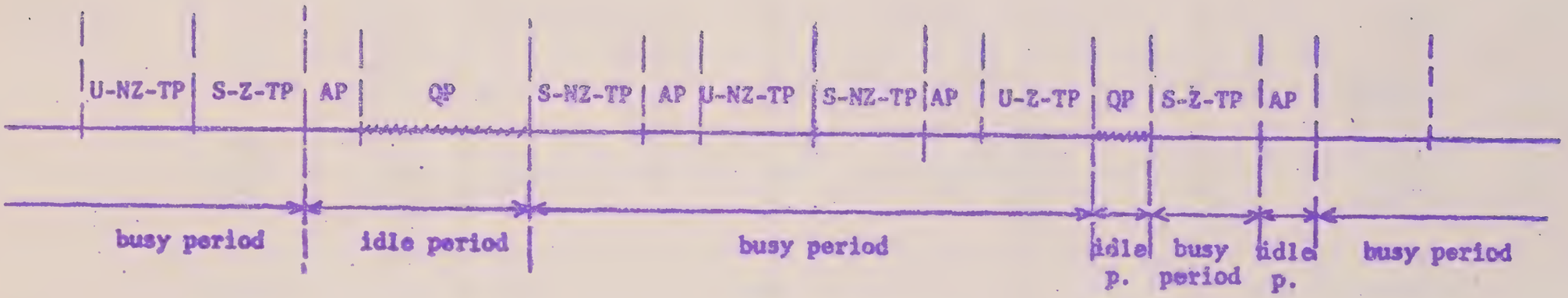
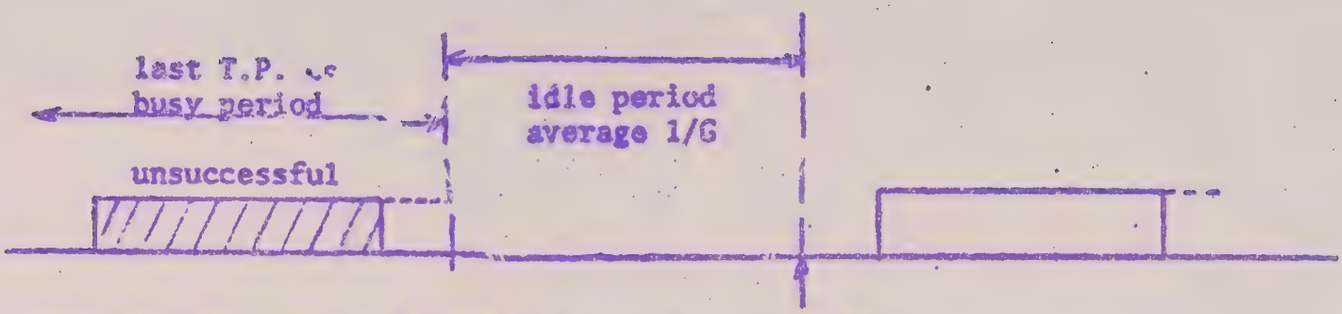
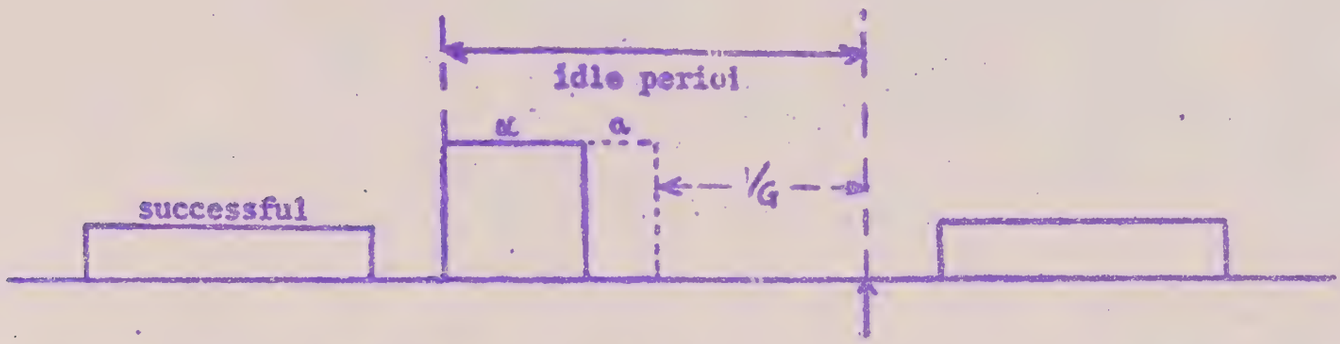


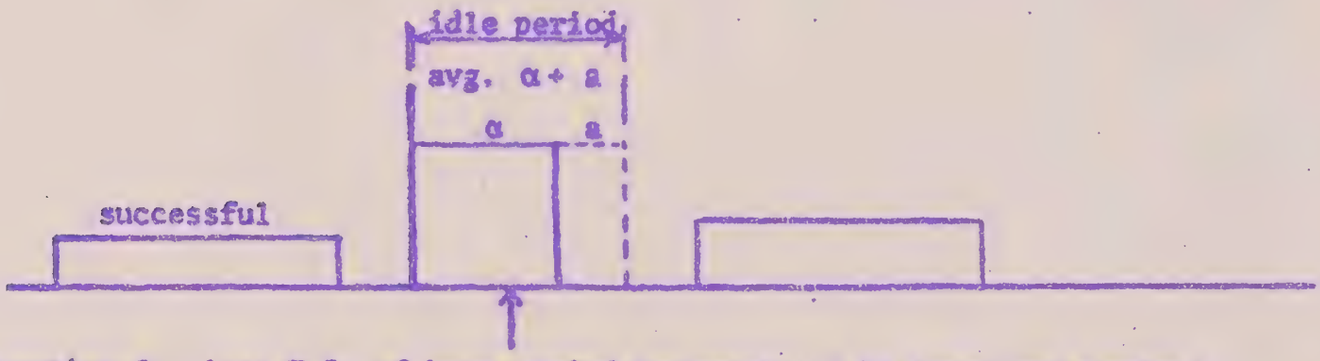
Figure 2. Busy and idle periods.



Situation 1: last T.P. of busy period unsuccessful.

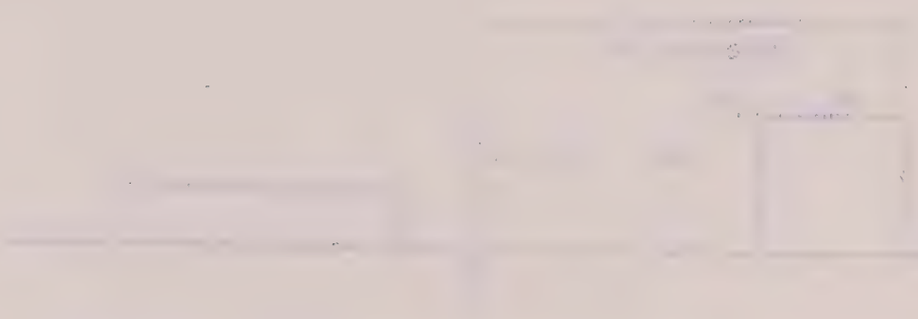


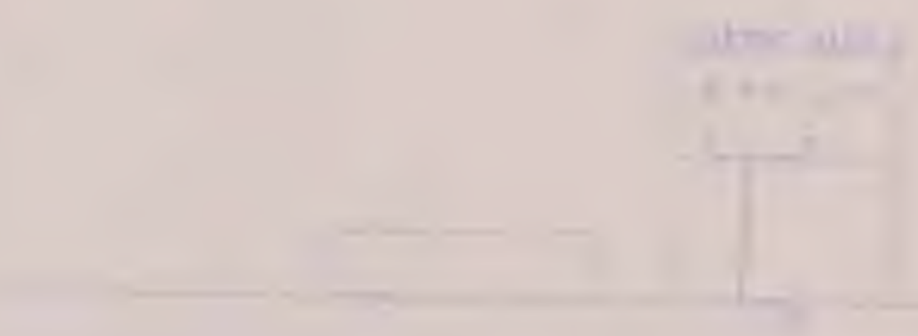
Situation 2: last T.P. of busy period successful - no arrivals during the ACK period.



Situation 3: last T.P. of busy period successful - at least one arrival during the ACK period.

Fig. 3. Various situations of idle periods.





K&E 5 X 5 TO ½ INCH 46 0860
7 X 10 INCHES MADE IN U.S.A.
KEUFFEL & ESSER CO.

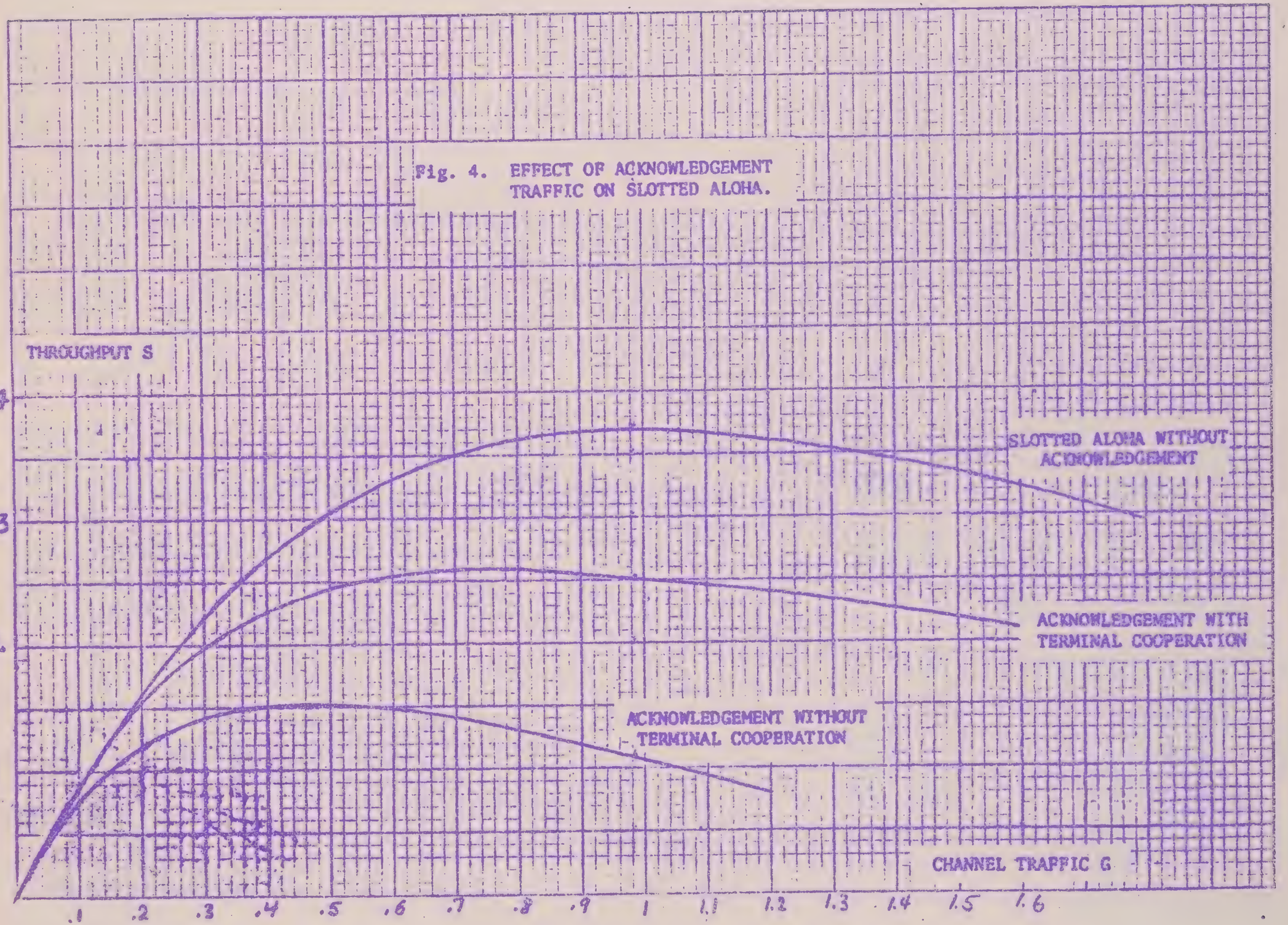
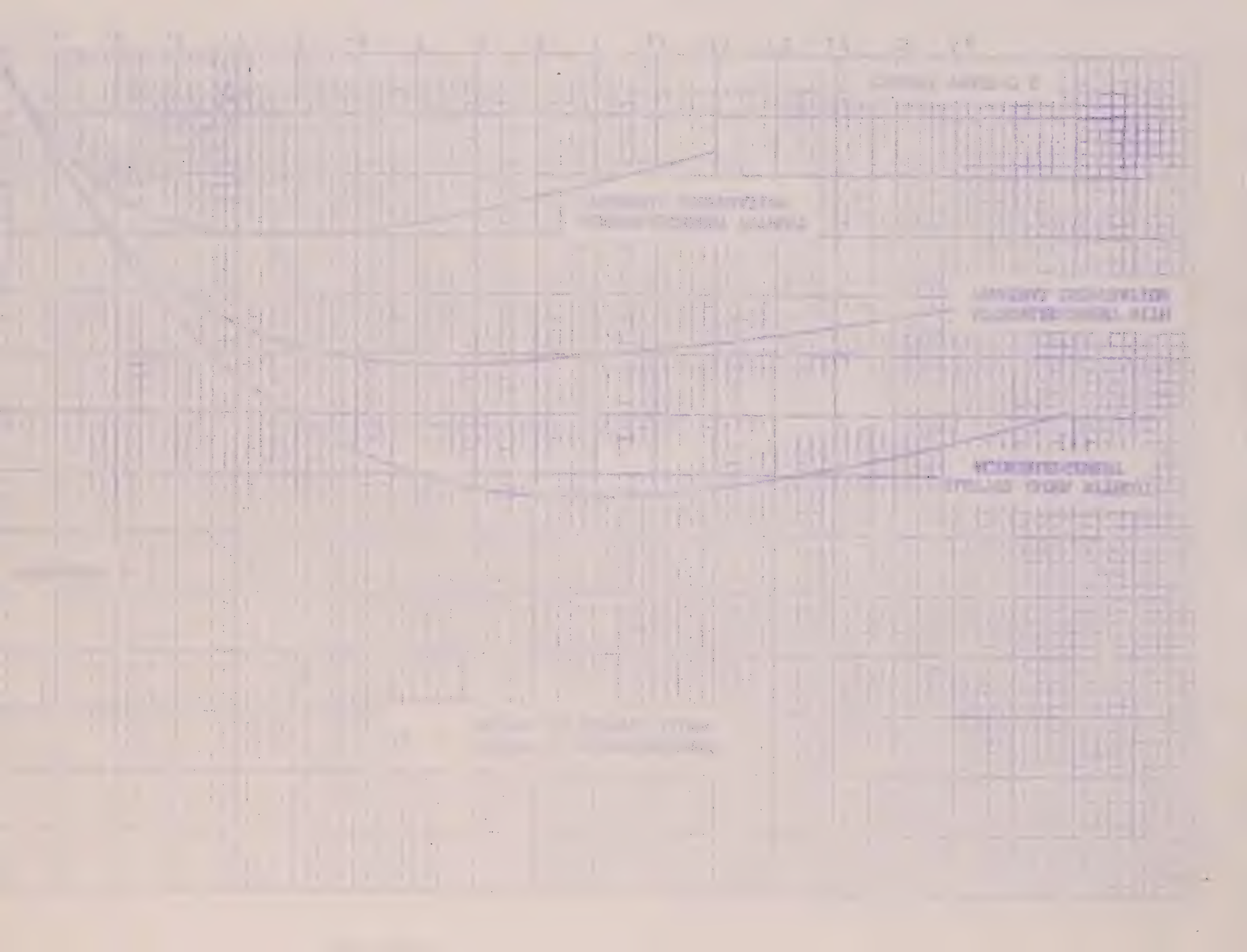
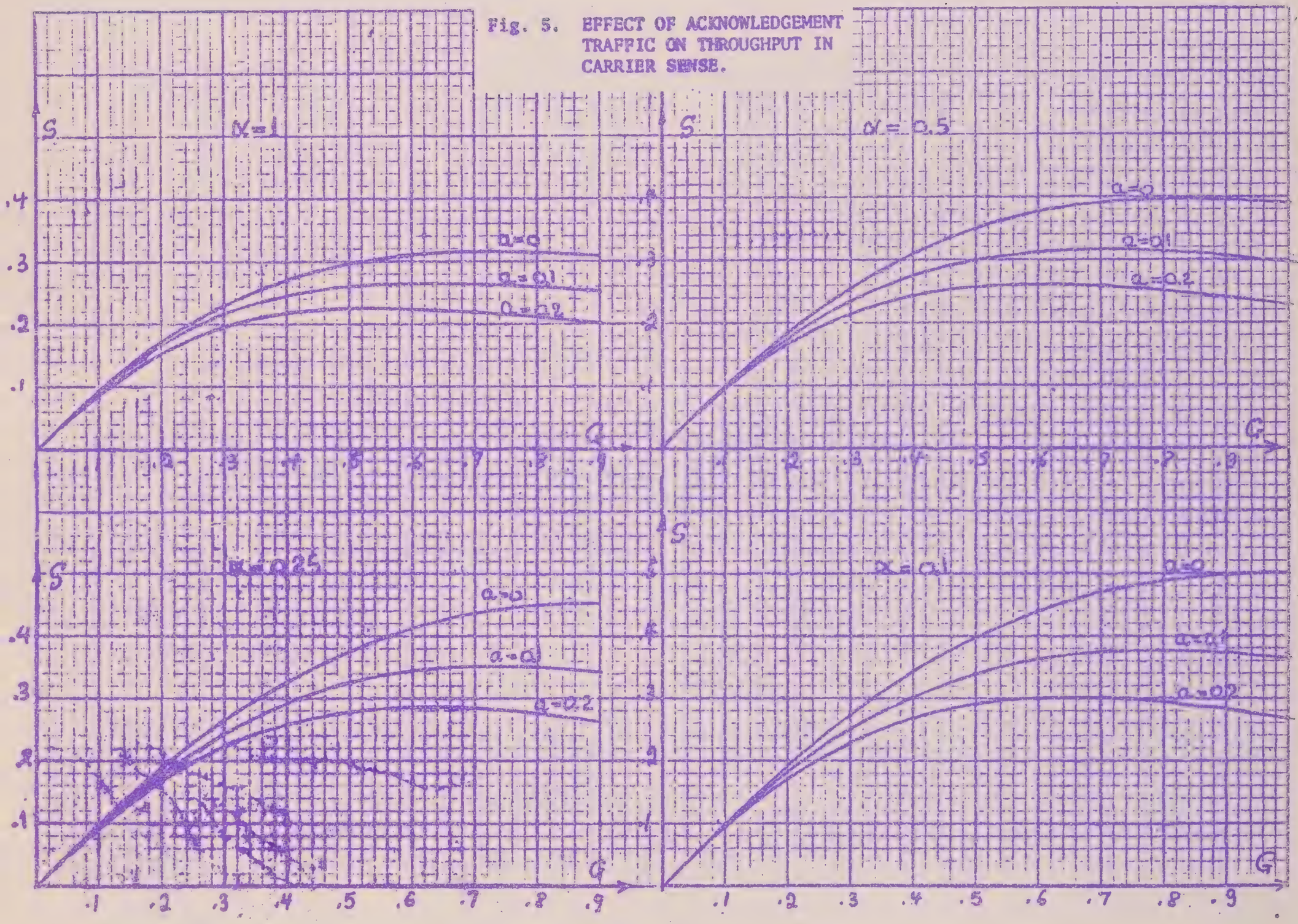


Fig. 4. EFFECT OF ACKNOWLEDGEMENT TRAFFIC ON SLOTTED ALOHA.



K&E 5 X 5 TO ½ INCH 7 X 10 INCHES 46 0860 MADE IN U.S.A.
KEUFFEL & ESSER CO.

Fig. 5. EFFECT OF ACKNOWLEDGEMENT TRAFFIC ON THROUGHPUT IN CARRIER SENSE.



K+E 5 X 5 TO ½ INCH 48 0550
7 X 10 INCHES MADE IN U.S.A.
KEUFFEL & ESSER CO.

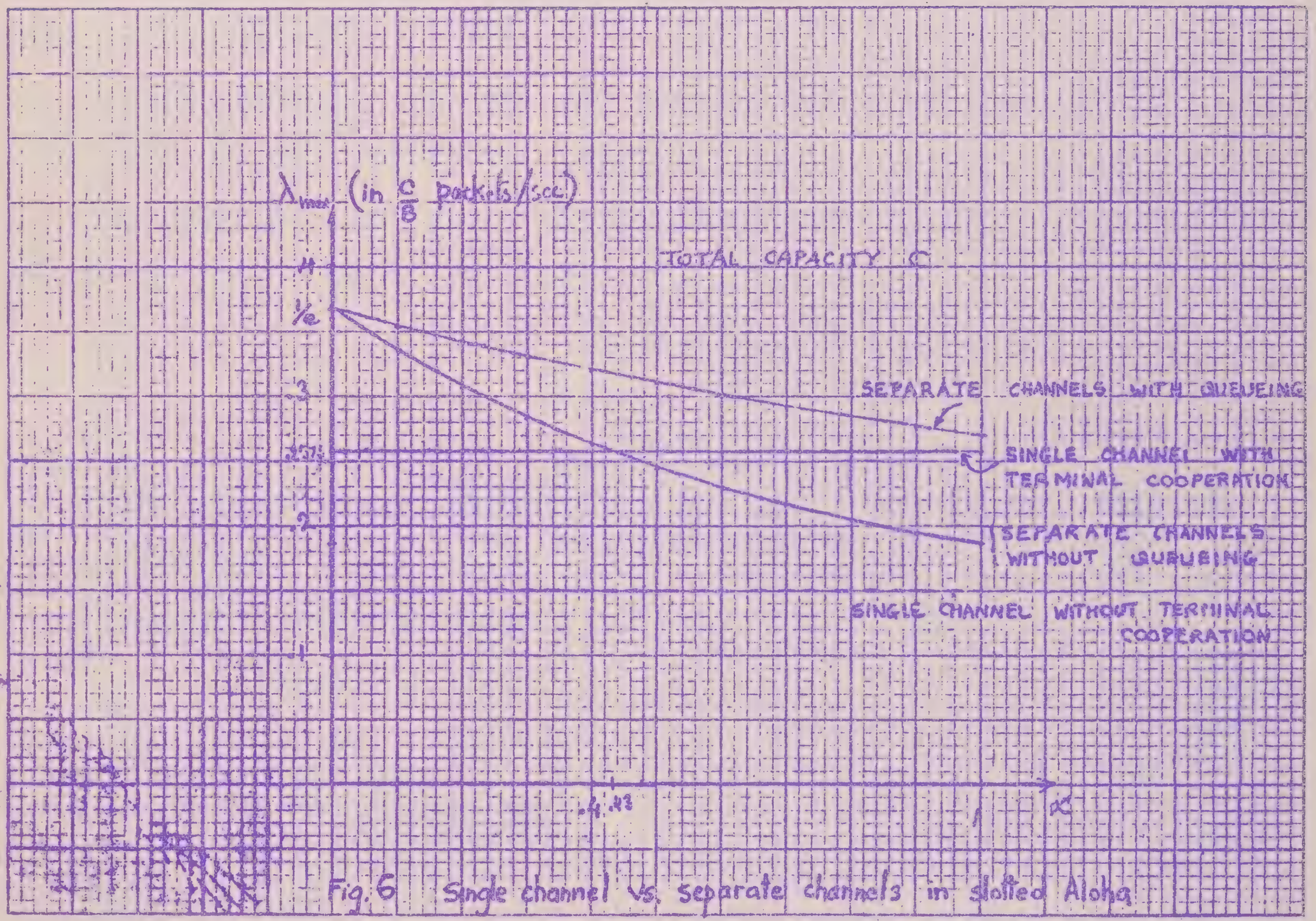
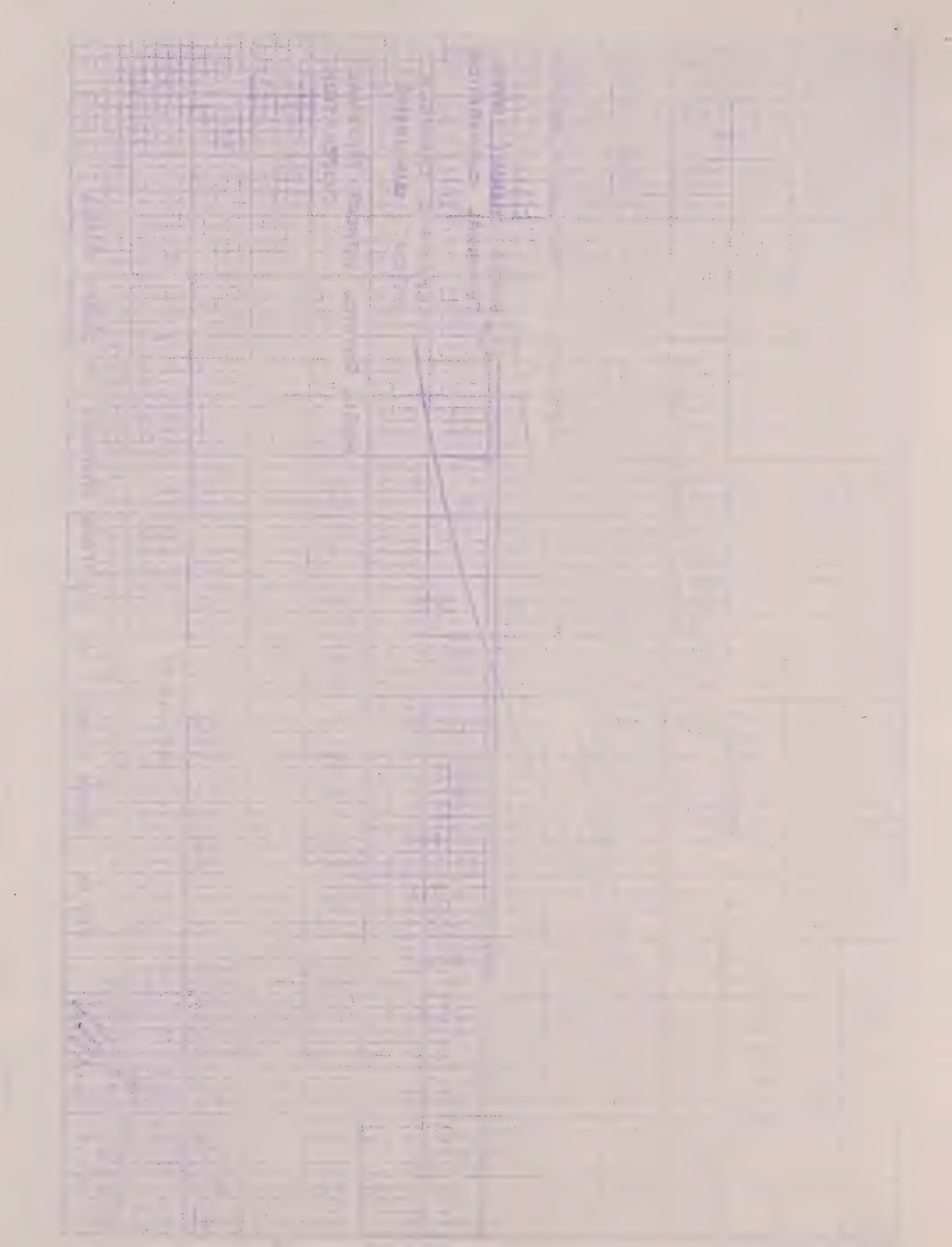


Fig. 6 Single channel vs. separate channels in slotted Aloha



K&E 5 X 5 TO ½ INCH 46 0860
7 X 10 INCHES MADE IN U.S.A.
KEUFFEL & ESSER CO.

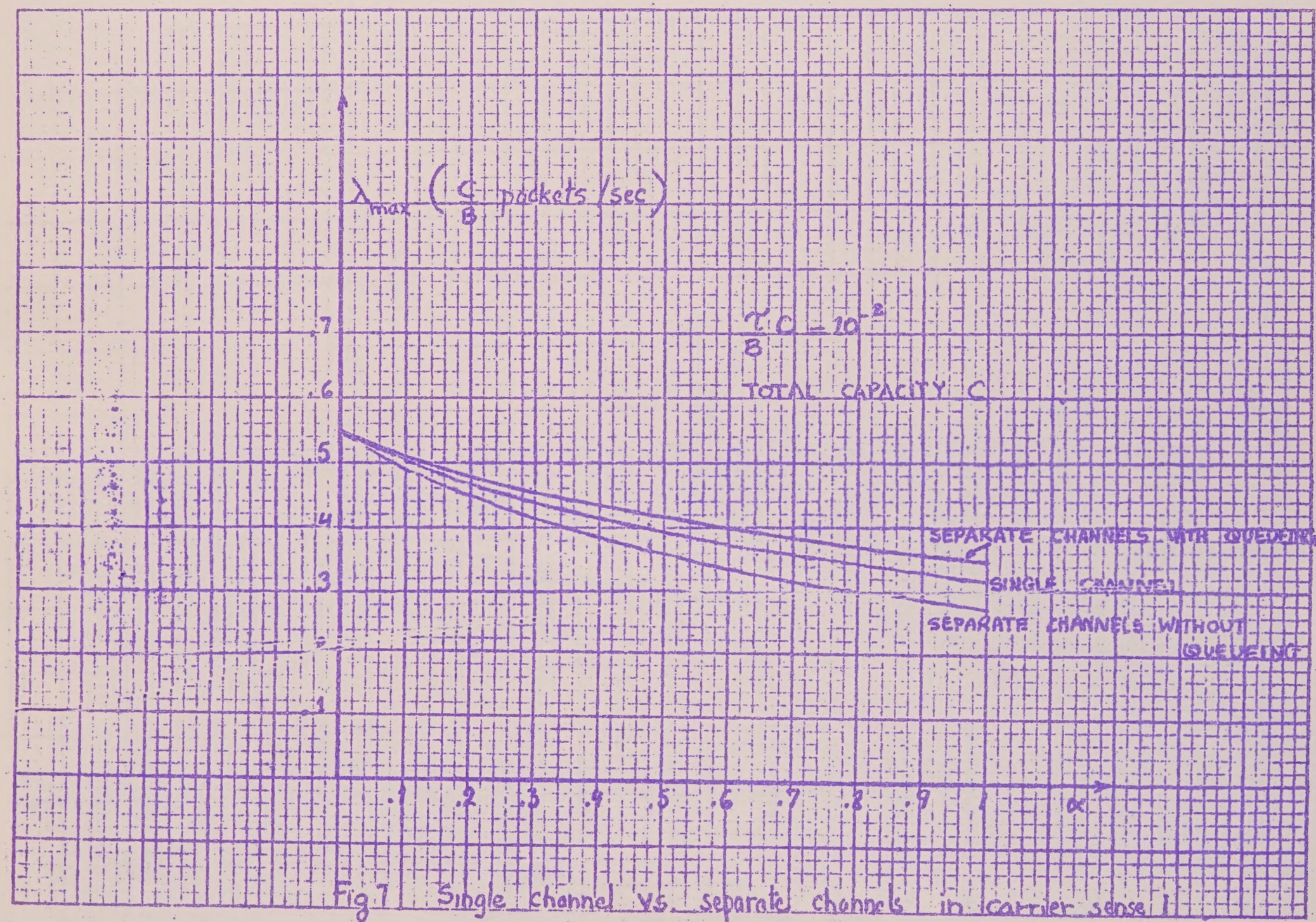
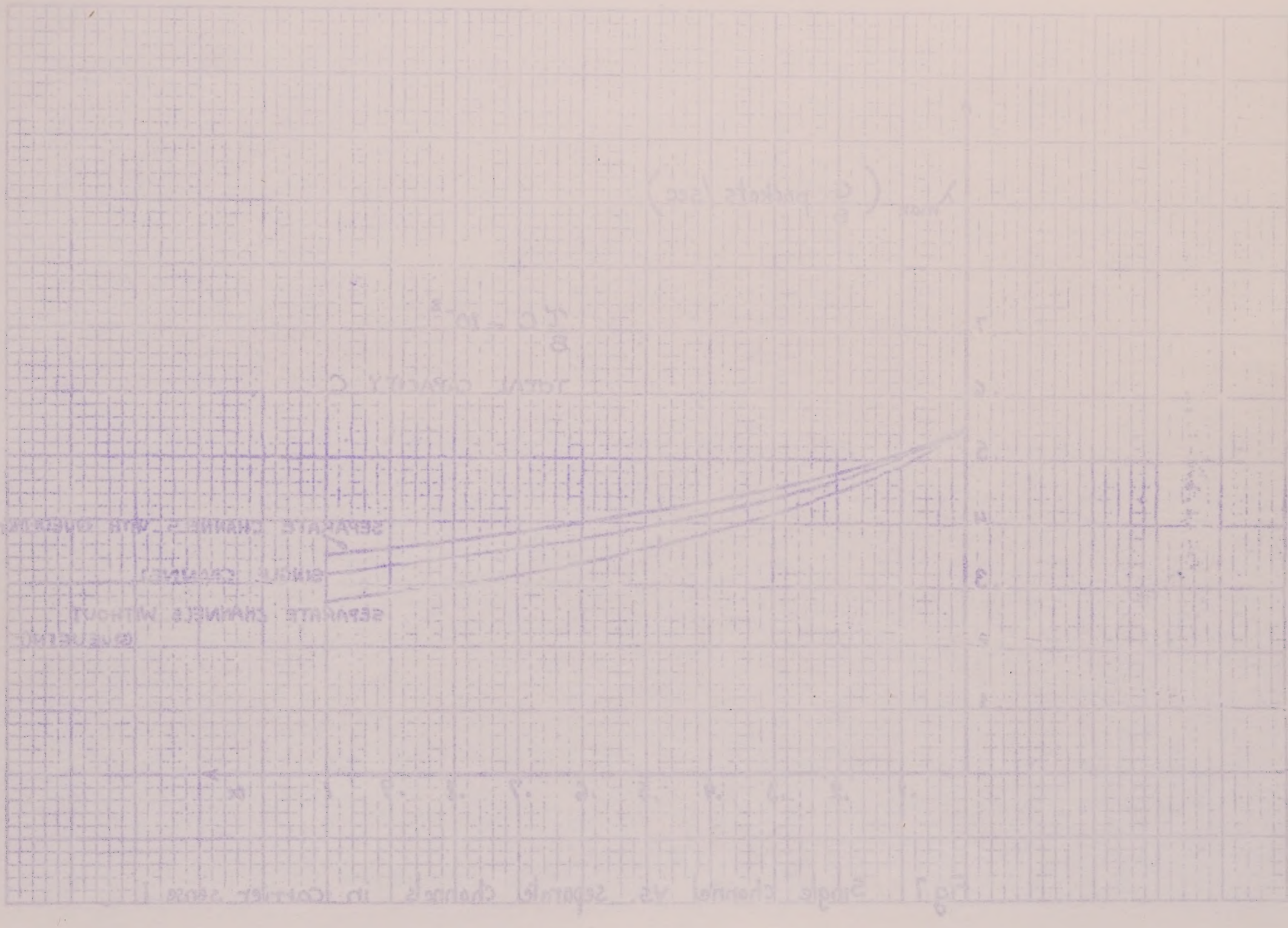


Fig 7 Single channel vs. separate channels in carrier sense



K·E 5 X 5 TO 7½ INCH 46 0860
7 X 10 INCHES MADE IN U.S.A.
KEUFFEL & ESSER CO.

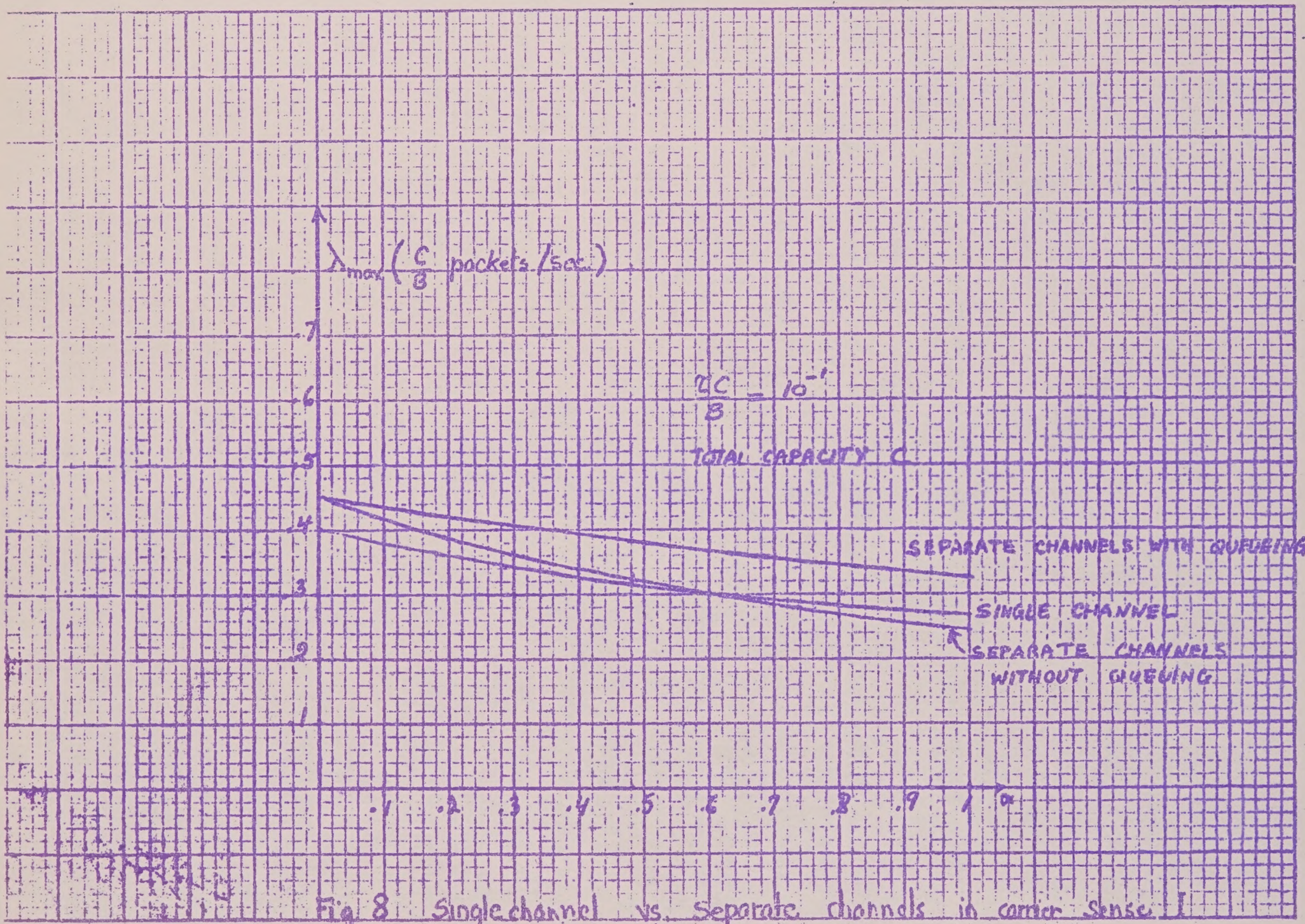
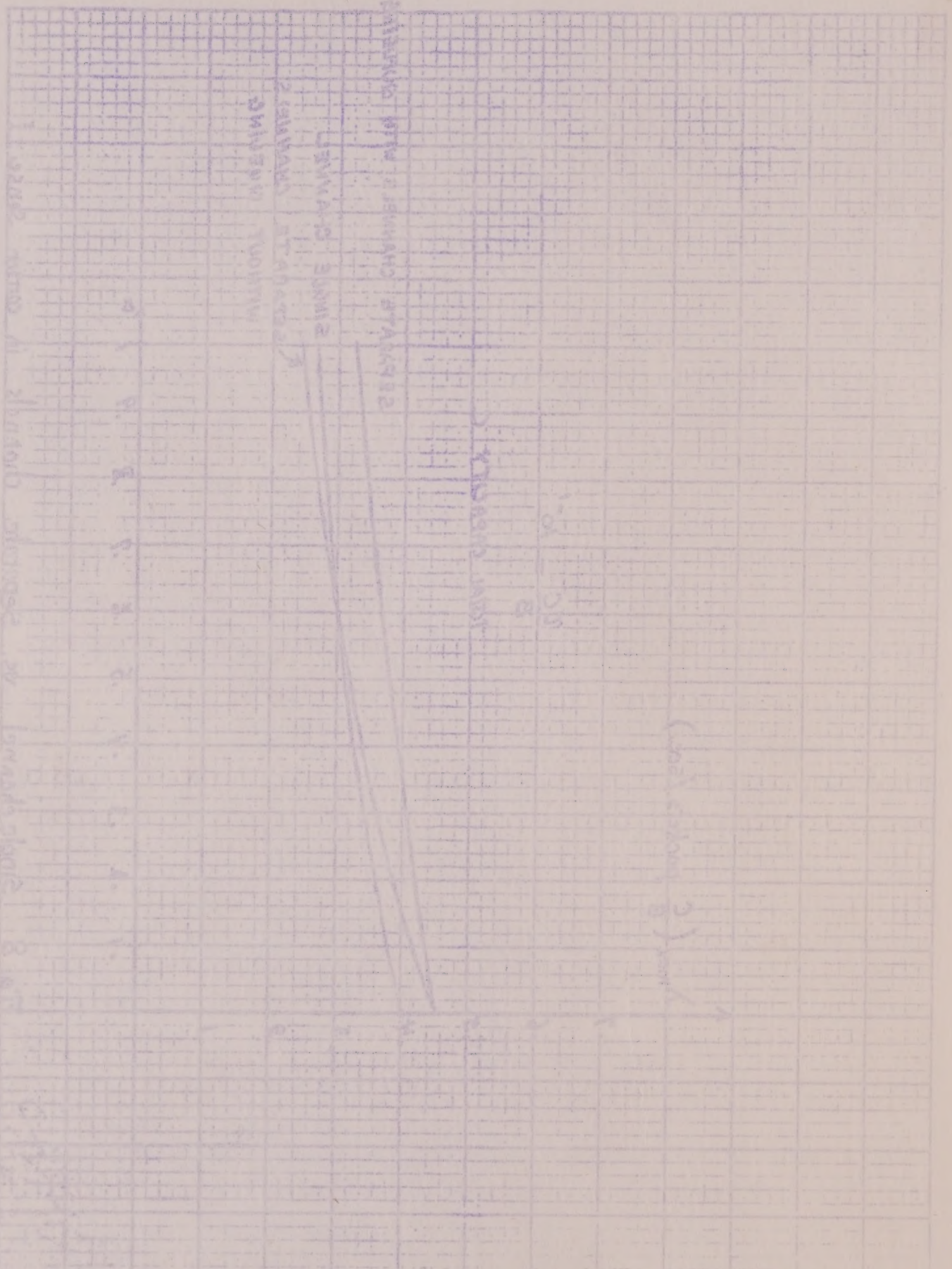



Fig 8 Single channel vs. Separate channels in carrier Sense